Solutions #1   Bergen County Math League   2018–2019

1–1. Let x be the area of the overlapping region. Then the difference in area between the non-overlapping regions is $(25\pi - x) - (16\pi - x) = 9\pi$.

1–2. The sum of the full set of 20 numbers is 120, and the sum of the remaining 17 numbers is 102. Thus $2x + 3x + 4x = 18 \Rightarrow x = 2$, so the numbers are $4, 6, 8$.

1–3. Let L be the length of each candle, and t the number of hours needed for the described situation to occur. At time t, the 4-hour candle has length $L - \frac{tL}{4}$, and the 3-hour candle has length $L - \frac{tL}{3}$. Therefore,

$$L - \frac{tL}{4} = 2\left(L - \frac{tL}{3}\right) \Rightarrow t = \frac{12}{5},$$

which is 2 hours and 24 minutes.

1–4. Let $x = 0.\overline{307692}$. Then $10^6 x = 307692 + x \Rightarrow x = \frac{307692}{999999} = \frac{4}{13}$.

1–5. Let a, b, and c be the side lengths, with c the hypotenuse. Then

$$a^2 + b^2 + c^2 = 2c^2 = 338 \Rightarrow c = 13,$$

so a and b are 5 and 12. The perimeter is $5 + 12 + 13 = 30$.

1–6. Create pairs of numbers as follows: $(0, 9999), (1, 9998), (2, 9997), \ldots, (9997, 2), (9998, 1), (9999, 0)$. The sum of the digits in each pair is 36. There are 1000 pairs with each integer appearing twice, so the sum we are looking for is

$$\frac{36 \cdot 1000}{2} = 180000.$$