Solutions #3   Bergen County Math League   2018–2019

3–1. $n^2 + 4n + 36 = (n + 2)^2 + 32$, so $(n + 2)^2$ must be a multiple of 32. But $32 = 2^5$ contains an odd number of factors of 2, so $n + 2$ must actually contain all 5 factors of 2. Therefore, $n = 30$.

3–2. If $m\angle A = x$, then $m\overset{\frown}{CB} = 2x$, and $m\overset{\frown}{CAB} = 3x$. Therefore, $5x = 360$.

3–3. $x^2 + y^2 = A$ and $(x + y)^2 = B$. Therefore,

$$B = (x + y)^2$$
$$= x^2 + 2xy + y^2$$
$$= A + 2xy$$

so $xy = \frac{B-A}{2}$.

3–4. The sum of the interior angles of an n-gon is $180(n - 2)$, and the sum of the exterior angles is 360. The polygon is regular, so

$$\frac{180(n - 2)}{n} = \frac{360}{n} + 100$$
$$\Rightarrow 180n - 360 = 360 + 100n$$
$$\Rightarrow n = 9$$

3–5. When Ace is 88, Speedy will be 44, since the sum of their ages will be 132 at that time. Therefore, $a = s + 44$. But $a + f + s = 73$, so $s + 44 + f + s = 73 \Rightarrow f = 29 - 2s$.

3–6. If the rightmost digit were 1, then moving to the left the digits would have to be at least 2 then 4, then 8, after which the process fails. Therefore, the rightmost digit has to be 0. If the next digit to the left were 2, then as before, moving to the left would yield digits of at least 3 and 6, and then the process fails again. Therefore, the number has to end with 10. Similarly, one can find that the next digit to the left has to be 2, so that the number ends with 210. From this point, trial and error yields 84210, 94210, and 95210.