Solutions #4 **Bergen County Math League** **2018–2019**

4–1. The distance between the midpoints is $\sqrt{3^2+3^2} = 3\sqrt{2}$, so the diagonal of the square is $6\sqrt{2}$. The area is then $\frac{1}{2}\left(6\sqrt{2}\right)^2 = 36$.

4–2. $4 \cdot 8^{10} = 2^{32}$, so its fourth root is $2^8 = 256$.

4–3. Let h, b, and d be the length of the head, body, and tail, respectively. Then $h = 9$, $b = 9 + t$, and $t = 9 + 0.5b$. Substitute the second equation into the third to get $t = 13.5 + 0.5t$, and solve to get $t = 27$. Then $b = 36$, so $h + b + t = 72$.

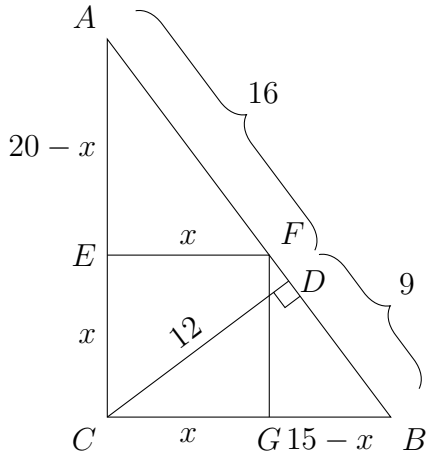
4–4. Start on the outside and work your way in.

$$\begin{aligned} &\log_{10}(\log_{10}(\log_{10}(\log_{10} x))) = 0 \\ \Rightarrow &\log_{10}(\log_{10}(\log_{10} x)) = 1 \\ \Rightarrow &\log_{10}(\log_{10} x) = 10 \\ \Rightarrow &\log_{10} x = 10^{10} \\ \Rightarrow &x = 10^{10^{10}} \end{aligned}$$

so $k = 10^{10}$.

4–5. n is a perfect cube, and since it is a three-digit number, its cube root is less than 10. Try $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$. Only $8^3 = 512$ works, with $5 + 1 + 2 = 8$.

4–6.



Since $\dfrac{AD}{CD} = \dfrac{CD}{DB}$, we get

$AD = 16, BD = 9, AC = 20, BC = 15.$

Now $\triangle AEF \sim \triangle FGB$, so

$$\frac{20 - x}{x} = \frac{x}{15 - x} \Rightarrow x = \frac{60}{7}$$