Solutions #5 **Bergen County Math League** **2018–2019**

5–1. The shortest distance is the greatest common divisor of 285 and 60, which is 15.

5–2. $1350 = 2 \cdot 3^3 \cdot 5^2$, so $n = 2^2 \cdot 5 = 20$.

5–3. Clearly $b = \pm 3$. If $b = 3$, then

$$(x^2 + ax + 3)^2 = x^4 + 2ax^3 + (a^2 + 6)x^2 + 6ax + 9 \Rightarrow a = -2.$$

If $b = -3$, then

$$(x^2 + ax - 3)^2 = x^4 + 2ax^3 + (a^2 - 6)x^2 - 6ax + 9$$

which is impossible. Therefore, $a = -2$ and $b = 3$.

5–4.

x	y	z	# possibilities
0	$0, \pm1, \pm2$	$0, \pm1, \pm2$	$1 \cdot 5 \cdot 5 = 25$
±1	$0, \pm1$	$0, \pm1, \pm2$	$2 \cdot 3 \cdot 5 = 30$
±1	±2	$0, \pm1$	$2 \cdot 2 \cdot 3 = 12$
±2	0	$0, \pm1, \pm2$	$2 \cdot 1 \cdot 5 = 10$
±2	±1	$0, \pm1$	$2 \cdot 2 \cdot 4 = 12$
±2	±2	0	$2 \cdot 2 \cdot 1 = 4$
			Total $= 93$

5–5. $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$. Therefore,

$$2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\Rightarrow \sin\left(\frac{A+B}{2}\right) = \cos\left(\frac{A+B}{2}\right) \quad \text{or} \quad \cos\left(\frac{A-B}{2}\right) = 0.$$

If $\sin\left(\frac{A+B}{2}\right) = \cos\left(\frac{A+B}{2}\right)$, then $\frac{A+B}{2} = \frac{\pi}{4} \Rightarrow A + B = \frac{\pi}{2}$. If $\cos\left(\frac{A-B}{2}\right) = 0$, then $A - B$ is an odd multiple of π, but this is impossible as A and B are acute angles.

5–6. At 12:00, the hour and minute hands are both pointing at the 12. Between 12:00 and 12:44, the minute hand moves $\frac{44}{60} \cdot 360° = 264°$, so it is $360° - 264° = 96°$ away from the 12. In the same time, the hour hand moves $\frac{1}{12}$ of this distance, or $\frac{264°}{12} = 22°$, so the angle between them is $96° + 22° = 118°$.