Solutions #6 **Bergen County Math League** **2018–2019**

6–1. $\frac{90540}{108648} = \frac{\cancel{4}\cdot 22635}{\cancel{4}\cdot 27162} = \frac{\cancel{9}\cdot 2515}{\cancel{9}\cdot 3018} = \frac{5\cdot\cancel{503}}{2\cdot 3\cdot\cancel{503}} = \frac{5}{6}$.

6–2. There are a total of $2^5 = 32$ possible outcomes for five tosses. Among these, $\binom{5}{3} = 10$ consist of three heads and two tails, which is the only way for Ace to net \$1. The probability of this occurring is therefore $\frac{10}{32}$, or $\frac{5}{16}$.

6–3. We need to minimize $D = |21r - 15b|$ for positive integers r, b. Note that $D = 3|7r - 5b|$ so that D is a multiple of 3, and that when $r = 2, b = 3$ we have $D = 3$, so this must be the minimum.

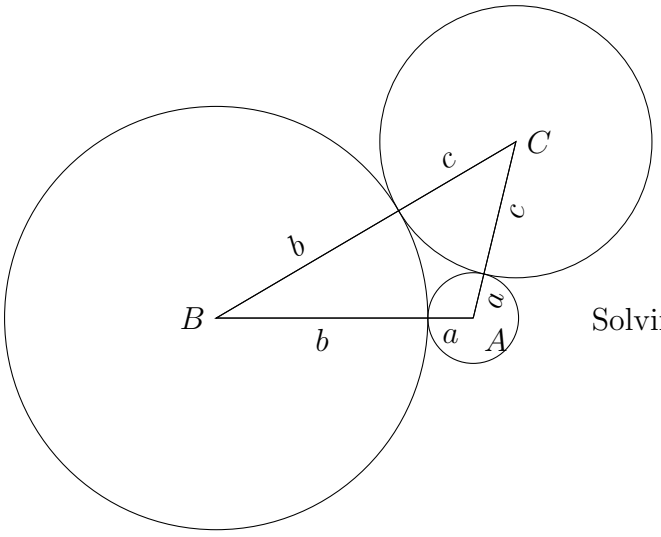
6–4. The total amount of money is always \$72. Work backwards.

	Player 1	Player 2	Player 3	
End of round 3	24	24	24	Player 1 loses
End of round 2	48	12	12	Player 2 loses
End of round 1	24	42	6	Player 3 loses
Start of game	12	21	39	

Speedy must be Player 3, so he started the game with \$39.

6–5. $\frac{1+\tan 70^\circ}{1-\tan 70^\circ} = \frac{\tan 45^\circ + \tan 70^\circ}{1 - \tan 45^\circ \tan 70^\circ} = \tan(45^\circ + 70^\circ) = \tan 115^\circ$

6–6.



$a + b = 17$
$b + c = 23$
$a + c = 12$

Solving yields $a = 3, b = 14, c = 9$.