Contest #1 **Bergen County Math League** **2019–2020**

Part I *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

1–1. The sides of a triangle are 15, 20, and 25. Find the length of the shortest altitude.

1–2. How many ordered pairs of integers (x, y) simultaneously satisfy $y = x-3$ and $x^2+y^2 \leq 25$?

Part II *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

1–3. How many positive integers less than 1 million are perfect squares or perfect cubes?

1–4. If $9 < y < 18$, find all pairs of integers (x, y) for which $\frac{1}{x} + \frac{1}{y} = \frac{1}{9}$.

Part III *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

1–5. A pizza shop sells pizzas in three shapes: circles with a 10-inch diameter, squares with 9-inch sides, and regular hexagons with 6-inch sides. If they are all the same price, which shape is the best buy?

1–6. Let f be a function for which $f(4x - 3) + xf(x^2) = 8x$ for all integers x.Find the numerical value of $f(-7)$.

Answers

1–1. 12

1–2. 6

1–3. 1089

1–4. $(36, 12), (90, 10)$

1–5. hexagon

1–6. -4