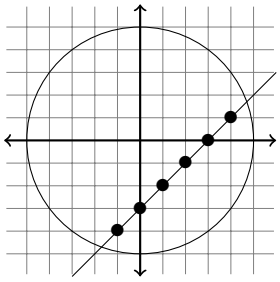
Solutions #1 **Bergen County Math League** **2019–2020**

1–1. This is a right triangle, so 15 and 20 are two of the altitudes. If the altitude to the hypotenuse has length a, then the area of the triangle can be calculated as $\frac{15\cdot 20}{2}$ or as $\frac{25a}{2}$, so $a = \frac{15\cdot 20}{25} = 12$.

1–2. 

1–3. First,

$$0 < x^2 < 10^6 \Rightarrow 0 < x < 10^3$$

so there are 999 perfect squares. Similarly,

$$0 < x^3 < 10^6 \Rightarrow 0 < x < 10^2$$

so there are 99 perfect cubes. However, any number that is a perfect 6th power is counted in both of these. As before,

$$0 < x^6 < 10^6 \Rightarrow 0 < x < 10$$

so there are 9 duplicates. The answer is therefore $999 + 99 - 9 = 1089$.

1–4. Solve for x:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{9} \Rightarrow 9y + 9x = xy \Rightarrow x = \frac{9y}{y-9} = 9 + \frac{81}{y-9}$$

The only values of y (between 9 and 18) for which this is an integer are 10 and 12. If $y = 10$ then $x = 9 + 81 = 90$, and if $y = 12$, then $x = 9 + 27 = 36$.

Alternatively, by trial and error simply substitute $y = 10, 11, 12, 13, 14, 15, 16, 17$.

1–5. The area of the circle is $25\pi \approx 78.5$. The area of the square is 81. The area of the hexagon is $\frac{3\sqrt{3}}{2} \cdot 6^2 \approx 93.5$.

1–6. First let $x = 1$: $f(1) + 1 \cdot f(1) = 8 \Rightarrow f(1) = 4$.

Now let $x = -1$: $f(-1) = f(-7) + -1 \cdot f(1) = -8$, so $f(-7) = -8 + f(1) = -4$.