Solutions #2 **Bergen County Math League** **2019–2020**

2–1. By definition this is 1 radian, so $\frac{180}{\pi}$ degrees.

Alternatively, let x be the degree measure of the angle. Then

$$2\pi r\left(\frac{x}{360}\right) = r \Rightarrow x = \frac{360}{2\pi} = \frac{180}{\pi}$$

2–2. If the expression is a perfect cube, it must factor as $(3x^2 + ax - 2)^3$. By the Binomial Theorem,

$$\left(3x^2 + (ax-2)\right)^3 = \left(3x^2\right)^3 + 3\left(3x^2\right)^2(ax-2) + \cdots = 27x^2 + 27ax^5 + \cdots$$

so $a = 4$, and the cube root is $3x^2 + 4x - 2$.

2–3. The area of the figure is 25. The shaded region consists of four triangles, each with base 1, two with height 2, and two with height 3, for a total area of 5.

Alternatively, the unshaded region consists of four triangles, each with base 4, two with height 2, and two with height 3, for a total area of 25.

2–4.

$$\left.\begin{aligned} x\left(x^2+y^2\right) &= 10 \\ y\left(x^2+y^2\right) &= 20 \end{aligned}\right\} \Rightarrow y = 2x$$

Substitute into the second equation and solve:

$$2x\left(x^2 + 4x^2\right) = 10x^3 = 20 \Rightarrow x = \sqrt[3]{2} \text{ and } y = 2\sqrt[3]{2}$$

2–5. $8^x + 8^{x+1} = 8^x + 8\cdot 8^x = 9\cdot 8^x$. Now $9\cdot 8^x = 144 \Rightarrow 8^x = 16$, or $2^{3x} = 2^4$, so $x = \frac{4}{3}$.

2–6. The first stone takes 6 feet, the second stone 12 feet, the third stone 18 feet, and so on, with the last stone taking 1200 feet. These distances form the arithmetic sequence $a_n = 6n$, with sum

$$200\left(\frac{6+1200}{2}\right) = 120600$$