Solutions #4 **Bergen County Math League** **2019–2020**

4–1. The area of the upper region is

$$\frac{1}{2}\pi(3.5x)^2 - \frac{1}{2}\pi(1.5x)^2 + \frac{1}{2}\pi(2x)^2 = 7\pi x^2,$$

and the area of the lower region is

$$\frac{1}{2}\pi(3.5x)^2 + \frac{1}{2}\pi(1.5x)^2 - \frac{1}{2}\pi(2x)^2 = \frac{21}{4}\pi x^2.$$

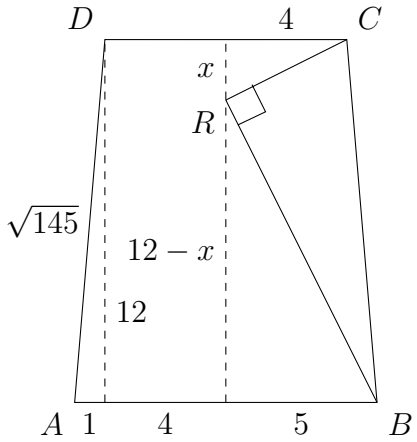
4–2. Set $y = x^2 + x$. Then $y^2 - 18y + 72 = 0 \Rightarrow (y-6)(y-12) = 0 \Rightarrow x^2 + x - 6 = 0$ or $x^2 + x - 12 = 0$. The former gives $x = -3, 2$ and the latter gives $x = -4, 3$.

4–3. By inspection, 1 through 6 can all occur ($3-2$, $5-3$, $5-2$, $7-3$, $7-2$, $11-5$). The difference of odd primes is even, and since 9 is not prime, 7 cannot occur as a difference.

4–4. Rewrite as $(2^x)^2 - \frac{9}{2}(2^x) + 2 = 0$. This is quadratic in 2^x and results in $2^x = \frac{1}{2}$ or 4, giving $x = -1, 2$.

4–5. Factor $6x^2 + 7ix + 3 = (3x - i)(2x + 3i)$, or use the quadratic formula with $b = 7i$.

4–6.



$CB^2 = AD^2 = 145$
$RC^2 + RB^2 = CB^2$
$(x^2 + 16) + [(12 - x)^2 + 25] = 145$
$x^2 - 12x + 20 = (x-10)(x-2) = 0 \Rightarrow x = 2, 10$