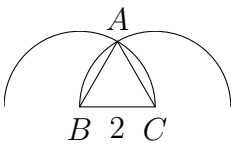
Solutions #5 **Bergen County Math League** **2019–2020**

5–1. The semi-perimeter of the triangle is $\frac{1}{2}(15+14+13)=21$, so Heron's formula gives the area as $\sqrt{21(21-15)(21-14)(21-13)}=\sqrt{7056}=84$.

5–2. There are a total of $\binom{24}{2}=276$ handshakes divided among 12 pairs of simultaneous handshakes, so they require $\frac{276}{12}\cdot 30=690$ seconds.

5–3. $f(x)=f\left(2\left(\frac{x}{2}\right)\right)=\left(\frac{x}{2}\right)^2-\frac{x}{2}+3$.

5–4.



$\triangle ABC$ is equilateral with area $\sqrt{3}$. Sector ABC is $\frac{1}{6}$ of a circle with area 4π, so its area is $\frac{2\pi}{3}$. Therefore, the area of segment AC is $\frac{2\pi}{3}-\sqrt{3}$. Finally, the desired area is

$$2\left(\frac{2\pi}{3}\cdot\sqrt{3}\right)+\sqrt{3}=\frac{4\pi}{3}-\sqrt{3}$$

5–5.

	Ace won		Flash won		Speedy won	
	Claim 1	Claim 2	Claim 1	Claim 2	Claim 1	Claim 2
Ace	F	T	T	F	T	T
Flash	F	F	T	F	T	T
Speedy	T	T	T	F	T	F

The only scenario in which the truth claims follow the pattern indicated is the one in which Ace won.

5–6. $1000001=10^6+1=\left(10^2\right)^3+1^3$. Now x^3+y^3 has $x+y$ as a factor, so one of the factors is 101. Divide to find the other factor.