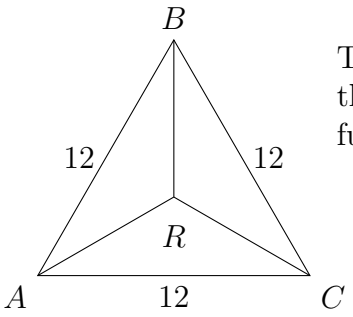
Solutions #6 **Bergen County Math League** **2019–2020**

6–1. $24x^2 + 49x - 40 = 24x^2 - 15x + 64x - 40 = 3x(8x-5) + 8(8x-5) = (8x-5)(3x+8)$.

6–2. Substituting $1-x$ for x gives $2f(1-x) + f(x) = (1-x)^2$. Subtract this from twice the original equation to get $3f(x) = 2x^2 - (1-x)^2$, giving $f(x) = \frac{x^2+2x-1}{3}$.

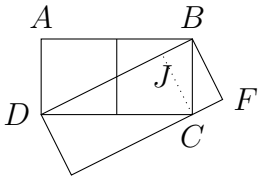
6–3.



The region R is precisely the bottom section of of the triangle, as drawn. Its area is $\frac{1}{3}$ the area of the full triangle,

$$\frac{1}{3} \cdot 36\sqrt{3} = 12\sqrt{3}$$

6–4.



The area of $\triangle BCD$ is 1, so $1 = \frac{1}{2} \cdot BD \cdot CJ = \frac{1}{2} \cdot \sqrt{5} \cdot CJ \Rightarrow CJ = \frac{2}{\sqrt{5}}$. Now $BF = CJ$, so the area of the slanted rectangle is $\frac{2}{\sqrt{5}} \cdot \sqrt{5} = 2$.

6–5. $x + y = xy \Rightarrow x = 1 + \frac{1}{y-1}$. This can only be an integer if $y = 2$, and then $x = 2$ as well.

6–6. Let S be the infinite sum. Then

$$S = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots$$

and

$$3S = 1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \frac{5}{81} + \cdots$$

Now subtract S from $3S$ to obtain

$$2S = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

This is a convergent geometric series with sum $\frac{1}{1-1/3} = \frac{3}{2}$, so $S = \frac{3}{4}$.