Bergen County Math League

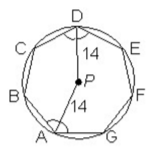
Contest #1 **2021** **Answers/Soluions**

1-1. **Answer:** 14

Let t_n denote the n^{th} term. It is given that $t_4 + t_5 = t_6$. It follows that $t_5 = t_6 - t_4 = 6 - 4 = 2$. Then, $t_7 = t_5 + t_6 = 2 + 6 = 8$. Thus, $t_8 = t_6 + t_7 = 6 + 8 = 14$.

1-2. **Answer:** 14

All the bisectors of the angles of any regular polygon meet at the center of that polygon's circumscribed circle. The length of $\overline{AP}$ is the length of a radius of the circumcircle, which is 14.



1-3. **Answer:** ½

There is $\frac{1}{2}$ liter remaining in the urn after the first pouring. Anytime there is $\frac{1}{2}$ liter in the first urn, and $\left(\frac{1}{n}\right)\left(\frac{1}{2}\right)$ liters is added to it, the number of liters then removed will be $\left(\frac{1}{n+1}\right)\left(\frac{n+1}{2n}\right) = \frac{1}{2n}$, the amount just added. Therefore, following the 1st pouring, there will always be $\frac{1}{2}$ liter in the first urn after a pouring is made from that urn. The 2021st pouring is made from the first urn, so the number of liters then remaining will be $\frac{1}{2}$.

1-4. **Answer:** 2

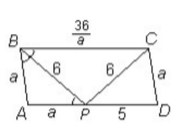
In an n-element set, there are 2^n subsets. If n is the number of elements in *A*, there are 2^nelements in set *B*. Therefore, *B* has 2^{2^n} subsets. Since $2^{2^n} = 16 = 2^{2^2}, n = 2$.

1-5. **Answer:** 0 or none

Since *x*, *y*, and *z* are unequal, the original expression has a sum of $\frac{x(z-y)+y(x-z)+z(y-x)}{(x-y)(y-z)(z-x)}$. This simplifies to $\frac{xz-xy+xy-yz+yz-xz}{(x-y)(y-z)(z-x)} = 0$, so the number of triples is 0 or none.

1-6. **Answer:** 4

Method I: If *AB = a*, then *AP=a* also. Since $\triangle\text{ABP} \sim \triangle\text{PBC}$, $BC = \frac{36}{a}$. Since $BC = AD, \frac{36}{a} = a + 5$. Consequently, $a^2 + 5a - 36 = (a-4)(a+9) = 0$, and $AB = a = 4$.



Method II: In $\triangle\text{PDC}$, $6^2 = 5^2 + a^2 - 10a\cos D$ and $\cos D = \frac{5^2+a^2-6^2}{10a}$. In $\triangle\text{PAB}$, $6^2 = a^2 + a^2 - 2a^2\cos A$, and $\cos A = \frac{2a^2-6^2}{2a^2}$. Since $\angle D$ is supplementary to $\angle A$, $\cos D = -\cos A$, so $\frac{25+a^2-36}{10a} = -\frac{2a^2-36}{2a^2}$. Clearing fractions, $a(a-4)(a^2+14a+45) = 0$. This equation has one positive solution, $a = 4$.