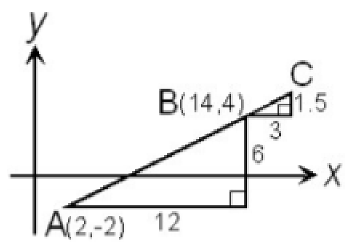
Bergen County Math League

Contest #3 **2021** **Answers/Solutions**

3-1. **Answer:** 10
Since $3! \times 5! = 6 \times 120 = 720 = 8 \times 9 \times 10, 7! \times (3! \times 5!) = 7! \times (8 \times 9 \times 10) = 10!$. Thus, n=10.

3-2. **Answer:** $\left(17, 5\frac{1}{2}\right)$
Using the similar triangles shown, the coordinates of point C are $\left(14 + \frac{1}{4}(12), 4 + \frac{1}{4}(6)\right) = \left(17, 5\frac{1}{2}\right)$.

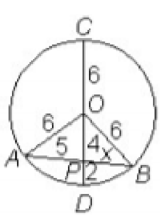


3-3. **Answer:** 45
Method I: A 30-minute period with 2 mouths eating is the equivalent to a 60-minute period with one mouth eating. The leaf was eaten in 2 stages: the 1st a 30-minute period with one mouth eating, and the 2nd a 30-minute period with 2 mouths eating. Therefore, the leaf could have been eaten by 1 mouth in $.30 + 60 = 90$ minutes. To eat half the leaf with only 1 mouth, the number of minutes required would have been 45.

Method II: Treat this as a work problem. If 1 mouth needs m minutes to eat 1 leaf, $\frac{2}{m}(30) + \frac{1}{m}(30) = 1$. Solving, $m = 90$, so half that time is 45 minutes.

3-4. **Answer:** 7
Let b be the base of the numeration system. The angles are supplementary, so $(b^2 + 4) + (2b^2 + 4b + 1) = 180$. Solving, the positive root is $b = 7$.

3-5. **Answer:** 4



Method I: Let O be the center of the circle. Since the length of a radius of the circle is 6, $CO = OD = 6$. Since $OP = 4, PD = 2$. In this circle, $AP \times PB = CP \times PD$. Thus, $5 \times x = 10 \times 2$, from which $x = 4$.

Method II: Since $m\angle APO + m\angle BPO = 180°$, we get $cos\angle APO = -cos\angle BPO$. By the law of cosines, we get $\frac{5^2+4^2-6^2}{2(5)(4)} = \frac{6^2-x^2-4^2}{2(x)(4)}$, or $x^2 + x - 20 = 0$, from which $x = 4$.

Method III: Draw altitude $\overline{OE}$ from O to $\overline{AP}$. Let $EP = y, EA = 5 - y$, and $OE = h$. Use the Pythagorean Theorem three times: In $\Delta OEA, h^2 + (5 - y)^2 = 6^2$; in $\Delta OEB, h^2 + (x + y)^2 = 6^2$; in $\Delta OEP, h^2 + y^2 = 4^2$. Subtracting the third equation from the first, $y = \frac{1}{2}$. Substituting into the second and third equations, and then subtracting the third equation from the second, we get $x^2 + x = 20$, from which $x = 4$.

3-6. **Answer:** $\left(\frac{2}{3}, \frac{5}{3}\right)$

Method I: Doubling both sides, $34x^2 - 20xy + 4y^2 - 12x + 4 = 0$. After regrouping, this equation becomes $(9x^2 - 12x + 4) + (25x^2 - 20xy + 4y^2) = 0$, from which $(3x-2)^2 + (5x-2y)^2 = 0$. The only solutions of this equation occur when both $3x - 2 = 0$ and $5x - 2y = 0$, so $(x, y) = \left(\frac{2}{3}, \frac{5}{3}\right)$.

Method II: Treat the equation as $2y^2 + (-10x)y + (17x^2 - 6x + 2) = 0$, a quadratic in y with $a = 2, b = -10x$ and $c = 17x^2 - 6x + 2$. By the quadratic formula, $y = \frac{10x \pm \sqrt{100x^2 - 4(2)(17x^2 - 6x + 2)}}{4}$. Since y is real, the discriminant must be non-negative, so $-4(3x-2)^2 \geq 0$. But this expression can't be positive, so $x = \frac{2}{3}$, and by substitution, $y = \frac{5}{3}$.