Bergen County Math League

Contest #1 **2021-2022** **Answers/Solutions**

1-1. **Answer:** 457

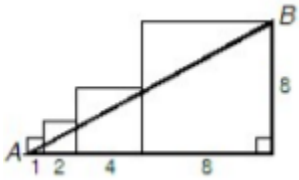
The product is 140, so let's factor: $140 = 2x2x5x7$. The only arrangement of factors into an increasing sequence of one-digit numbers is $4x5x7$, so my street address is 457.

1-2. **Answer:** 2028

The roots are integers, so the left side factors. Since there's only one positive value of a, 2027 must be prime. Factoring, $(x + 2027)(x + 1) = 0$. Multiplying, $x^2 + 2028x + 2027 = 0$, so $a = 2028$.

1-3. **Answer:** 17

Method I: In the diagram, there's a right triangle with hypotenuse $\overline{AB}$ and legs 8 and $(1 + 2 + 4 + 8) = 15$. By the Pythagorean Theorem, $AB = 17$.



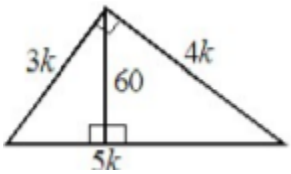
Method II: Coordinatize. Using $A(0,0)$ and $B(15,8)$, we can use the distance formula to get $AB = 17$.

[**NOTE:** The upper-left hand vertices of these squares are collinear, as are the upper right-hand vertices.]

1-4. **Answer:** 0

Make a list: $1^3 = 1$; $2^3 = 8$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; and $6^3 = 216$. Differences between cubes get even larger, so no two can differ by 64, and the answer is 0.

1-5. **Answer:** 3750



To maximize the triangle's area, let 60 be the length of the *shortest* altitude. The shortest altitude is drawn to the longest side, the hypotenuse. Since $(60)(5k) = (3k)(4k)$, $k = 25$. The area of the triangle is $150k = 150(25) = 3750$.

1-6. **Answer:** 4608

Method I: For each subset ***S*** that contains any given number, there's a subset ***S'***, called the complement of ***S***, that does not contain that number; so each number appears in just half the subsets. Since each number occurs $\frac{1}{2}x256 = 128$ times, the required sum is $128x(1 + 2 + \ldots + 8) = 4608$.

Method II: The subset with all 8 numbers as elements has an element sum of $1 + 2 + \ldots + 8 = 36$, so the average subset has an element sum half as great, 18. Finally, $256x18 = 4608$.

Method III: On average, there are 4 elements per subset; and the average value of an element is $4\frac{1}{2}$. The average sum is $4x4\frac{1}{2} = 18$, and $256x18 = 4608$.

Method IV: Each number appears in just half of the subsets (see Method I). The total of the sums equals the total of the sums of 128 sets, each containing all the numbers; and $128x(1 + 2 + \ldots + 8) = 4608$.