Bergen County Math League

Contest #4 **2021-2022** **Answers/Solutions**

4-1. **Answer:** 178

To maximize the largest angle, minimize the smallest two angles. If the two smallest angles have measures of 1 and 1, the measure of the largest angle of the triangle will be 178 or 178°.

4-2. **Answer:** 1681

Method I: From the binomial theorem, $\left(\sqrt{2}-1\right)^5 = \left(\sqrt{2}\right)^5 - 5\left(\sqrt{2}\right)^4 + 10\left(\sqrt{2}\right)^3 - 10\left(\sqrt{2}\right)^2 + 5\sqrt{2} - 1 = 4\sqrt{2} - 5(4) + 10\left(2\sqrt{2}\right) - 10(2) + 5\sqrt{2} - 1 = 29\sqrt{2} - 41 = \sqrt{1682} - \sqrt{1681}$, so $k = 1681$.

Method II: Use the table feature on your calculator, using $y = \sqrt{x+1} - \sqrt{x}$. Use "ask" for the "independent variable" and "auto" for the "dependent variable." By underestimating and overestimating, you should be able to zero in on x = 1681 in not too many tries.

4-3. **Answer:** 4042

Method I: By inspection, the sequence is *a*, *a*, 2*a*, 4*a*, 8*a*, …, so for *n* > 2, each term is twice the preceding term. Therefore, if the 100th term is 2021, the 101st term is 4042.

Method II: We'll let S_n represent the sum of the first *n* terms of the sequence, and a_n represent its *n*th term. We're told that $a_n = S_{n-1}$, so $2021 = a_{100} = S_{99}$, so $a_{101} = a_1 + \cdots + a_{99} + a_{100} = S_{99} + a_{100} = 2021 + 2021 = 4042$.

4-4. **Answer:** 20

In each isosceles triangle, let each vertex angle be *x*° and let each base angle be *y*°. In each triangle, *x*° + 2*y*° = 180°. In the diagram, the 12 triangles surround the center, so 10*x*° + 2*y*° = 360°. Subtracting the first equation from the second, 9*x*° = 180°, so *x*° = 20°.

4-5. **Answer:** 2022

The first output is $\frac{1492}{2022}$. This output is fed back in and the second output is 1492 divided by $\frac{1492}{2022}$. That output is 2022.

4-6. **Answer:** 576

Each pair (m, n) has the form (t^2, t^3), where *t* is a positive integer. Since $m + n = t^2 + t^3 = t^2(t+1)$ is a square, $t + 1$ must also be a square. Let $t + 1 = k^2$. [Conversely, if $m = t^2$ and $n = t^3$, where $t = k^2 - 1$, then $m + n = t^2 + t^3 = (1+t)(t^2) = k^2(k^2-1)^2 = (k^3 - k)^2$ is a square.] Since $m = t^2, t = k^2 - 1$, and $m < 1000$, it follows that $(k^2-1)^2 < 1000$. Taking square roots, $k^2 - 1 < 31.6\ldots$, so $k^2 < 32.6\ldots$. The largest such k^2 is 25, so $t + 1 = k^2 = 25$ and $t = 24$. Finally, $m = t^2 = 24^2 = 576$.

[Note: $m + n = 24^2 + 24^3 = 576 + 13824 = 14400 = 120^2$.]