Bergen County Math League
Calculators Permitted

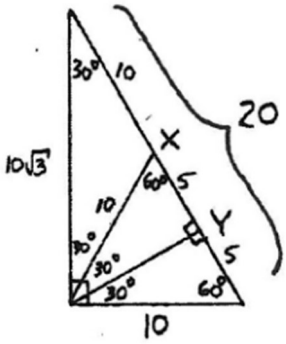
Contest #1 **2022-2023** **Answers/Solutions**

1-1. **Answer:** 264
There are $\binom{12}{2} = 66$ pair of teams, each of which plays four games.

1-2. **Answer:** 199
Since $f(n) = 2f(n-1) - f(n-2)$, it follows that $f(n-1) = \frac{f(n-2)+f(n)}{2}$, the average of $f(n-2)$ and $f(n)$. Therefore, $f(100)$ is the 100th odd positive integer, namely 199.

1-3. **Answer:** $\sqrt[4]{3}$
Let x be the side length of the cube. The diagonal of a face is $x\sqrt{2}$, so a diagonal of the cube is $\sqrt{\left(x\sqrt{2}\right)^2 + x^2} = x\sqrt{3}$. The volume is x^3, so $x^3 = x\sqrt{3}$, so $x = \sqrt[4]{3}$.

1-4. **Answer:** 5



The diagram is self-explanatory. Clearly, $\overline{xy} = 5$.

1-5. **Answer:** 6

CBA
+ABC
827

From the ones column, $A + C = 7$ or 17, so from the hundreds column it is clear that $A + C = 7$. Thus, $B + B = 2B = 10 + 2$ (for a carry of "1" into the hundreds column), so $B = 6$.

1-6. **Answer:** 15

$\frac{16b}{2} + \frac{3a}{2} = Area = \frac{ab}{2} = 150$. Thus, $\frac{16b}{2} + \frac{3a}{2} = 150$. But $b = \frac{300}{a}$; and thus $\frac{16}{2}\left(\frac{300}{a}\right) + \frac{3a}{2} = 150$. $\frac{2400}{a} + \frac{3a}{2} = 150$. So, $a = 20, b = 15$ (since both are integers).