Bergen County Math League
NO Calculators

Contest #2 **2022-2023** **Answers/Solutions**

2-1. **Answer:** 2π

The large circle has radius 3, so its area is 9π. The small circles each have an area of π, and there are 7 of them.

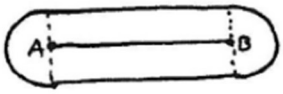
2-2. **Answer:** 44.5 or exact equivalent

$\cos x = \sin(90^\circ - x)$, so $\cos^2 x + \cos^2(90^\circ - x) = 1$. Arrange the terms in complementary pairs: $(\cos^2 1^\circ + \cos^2 89^\circ) + (\cos^2 2^\circ + \cos^2 88^\circ) + \cdots$. There are 44 such pairs, with $\cos^2 45^\circ$ and $\cos^2 90^\circ$ left unmatched. The sum is therefore $44 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0^2 = 44.5$.

2-3. **Answer:** $\frac{4}{3}$

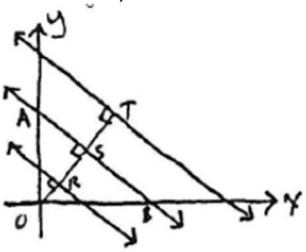
$a_1 = x^{\frac{1}{3}}$, $a_3 = x^{\frac{1}{2}}$. Therefore, $\frac{a_2}{x^{\frac{1}{3}}} = \frac{x^{\frac{1}{2}}}{a_2}$. So, $a_2^2 = x^{\frac{5}{6}}$, and $a_2 = \pm x^{\frac{5}{12}}$. Therefore, $r = \frac{\pm x^{\frac{5}{12}}}{x^{\frac{1}{3}}} = \pm x^{\frac{1}{12}}$. Now $a_{13} = a_1 r^{13-1} = \left(x^{\frac{1}{3}}\right)\left(\pm x^{\frac{1}{12}}\right)^{12} = \left(x^{\frac{1}{3}}\right)(x) = x^{\frac{4}{3}}$ and $k = \frac{4}{3}$.

2-4. **Answer:** 9π+60



$Area = (10)(6) + 2\left(\frac{1}{2}\right)(\pi)(3^2) = 9\pi + 60.$

2-5. **Answer:** 2, 22



$(AO)(OB) = (AB)(OS)$

$(3)(4) = (5)(OS)$, so $OS = \frac{12}{5}$

Therefore, $OR = \frac{2}{5}$, $OT = \frac{22}{5}$.

2-6. **Answer:** 140

From the lcm, we know that the other number must contain $(2^2)(7)$ as a factor. From the gcd, we know 5 is also a factor. Finally, $(2^2)(5)(7) = 140$ checks.
Alternatively, $(90)(x)$ =(lcm)(gcd). Therefore, $x = 140$. This last statement is a nice theorem from number theory.