Bergen County Math League
NO Calculators

Contest #4 **2022-2023** **Answers/Solutions**

4-1. **Answer:** $\left(2, \frac{5}{7}\right)$

Start with $x^3 - 3x^2y - xy + 2x - 2 = 0$, and multiply both sides by y to get $x^3y - 3x^2y^2 - xy^2 + 2xy - 2y = 0$. So now $x^3y - 3x^2y^2 - xy^2 + 2xy - 2y + x - 2 = x - 2 = 0$, and $x = 2$.

Then, $x^3 - 3x^2y - xy + 2x - 2 = 0 \Leftrightarrow 8 - 12y - 2y + 4 - 2 = 0 \Rightarrow y = \frac{5}{7}$. Finally, $(x, y) = \left(2.\frac{5}{7}\right)$.

4-2. **Answer:** 10^o or 50^o

$\sin(3x) = \frac{1}{2} \Rightarrow 3x = 30^o$ or 150^o. So, $x = 10^o$ or 50^o

4-3. **Answer:** 1

Since $f(3) = 9 = 4 + 5$, $a(3)^5 + b(3)^3 + c(3) = 4$.

Since $a(-3)^5 + b(-3)^3 + c(-3) = -[a(3)^5 + b(3)^3 + c(3)] = -4$, $f(-3) = -4 + 5 = 1$.

4-4. **Answer:** $(8, 6, 1)$ and $(9, 5, 2)$

The value of ABC is $100A + 10B + C$, and the value of CBA is $100C + 10B + A$. If they are both divisible by 7, then so is there a difference, which is $99(A - C)$. Now 99 is not divisible by 7, and 7 is prime, so $(A - C)$ must be divisible by 7. The only pairs (A, C) for which this is true are $(7, 0)$, $(8, 1)$, and $(9, 2)$. The first pair $(7, 0)$ does not work since then CBA would not be a three-digit number.

$A = 8, C = 1 \Rightarrow B = 6$ and $A = 9, C = 2 \Rightarrow B = 5$.

4-5. **Answer:** $32\frac{8}{11}$

Let $x =$ the distance the hour hand has moved, in minutes. Then since 30 minutes $= 180^o$, the minute hand must have moved $30 + x$ minutes. The minute hand moves 12 times as fast as the hour hand, so we get the equation $30 + x = 12x$. Solving, $x = \frac{30}{11} = 2\frac{8}{11}$, so the minute hand moved $x + 30 = 32\frac{8}{11}$ minutes.

4-6. **Answer:** $\frac{5}{9}$ or exact equivalent

The total number of outcomes is 6^3, and the number of outcomes with distinct values is $\binom{6}{3} \cdot 3! = 6 \cdot 5 \cdot 4$, so the answer is $\frac{6 \cdot 5 \cdot 4}{6 \cdot 6 \cdot 6} = \frac{5}{9}$.