Bergen County Math League
Calculators Permitted

Contest #5 **2022-2023** **Answers/Solutions**

5-1. **Answer:** -105
The smallest $|4 + x|$ can be is 0, so that $|5 + y| \leq 100 \Rightarrow -105 \leq y \leq 95$.

5-2. **Answer:** 1
Using $Area = \frac{1}{2}ab \sin C$, with $a = 1, b = 2, \Rightarrow$ max area when $m\angle C = 90^{\circ}$. Further, in this case, $c = \sqrt{5}$, and $2 \leq \sqrt{5} \leq 3$, as required. Thus, $Area = \frac{1}{2}(1)(2)(1) = 1$.

5-3. **Answer:** 84
The semi-perimeter of the triangle is $\frac{1}{2}(15 + 14 + 13) = 21$, so Heron's formula gives the area as

$$\sqrt{21(21-15)(21-14)(21-13)} = \sqrt{21(6)(7)(8)} = 84.$$

5-4. **Answer:** 27
Since $5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 = 81$, and since there are 3 rows having equal sums, the sum of the entries in any row must be $\frac{81}{3} = 27$.

5-5. **Answer:** 20
The maximum occurs when each circle intersects every other circle twice. Then for five circles, the number of intersections will be $0 + 2 + 4 + 6 + 8 = 20$, or $2\binom{5}{2} = 20$.

5-6. **Answer:** $t = \frac{2f}{f+1}$ or equivalent EQUATION solved for t
Let x be the common length of the candles, and let r be the rate, in length per hour, at which a candle burns. Then $r_{fat} = \frac{x}{f}$, and $r_{thin} = \frac{x}{t}$. After one hour, the lengths remaining in the fat and thin candles are $x - \frac{x}{f}$ and $x - \frac{x}{t}$, respectively. So $x - \frac{x}{f} = 2\left(x - \frac{x}{t}\right)$, and $t = \frac{2f}{f+1}$.