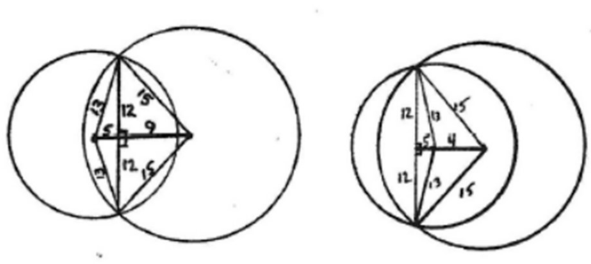
Bergen County Math League
NO Calculators

Contest #6 **2022-2023** **Answers/Solutions**

6-1. **Answer:** 14 and 4



From the diagrams, the two possible are $9+5$ or $9-5$, that is $14, 4$.

6-2. **Answer:** $\frac{1}{2}$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{4\sin x}{\cos x} + 1 = 0 \iff \sin^2 x - 4\sin x\cos x + \cos^2 x = 0$$

$$\iff 1 - 4\sin x\cos x = 1 - 2(\sin 2x) = 0$$

$$\iff \sin 2x = \frac{1}{2}$$

6-3. **Answer:** 7

$$x - \frac{1}{x} = 1 \implies \left(x - \frac{1}{x}\right)^2 = 1^2 \iff 1 = x^2 - 2 + \frac{1}{x^2} \iff x^2 + \frac{1}{x^2} = 3$$

But, $\left(x^2 + \frac{1}{x^2}\right)^2 = 3^2 \Rightarrow x^4 + 2 + \frac{1}{x^4} = 9 \iff x^4 + \frac{1}{x^4} = 7.$

6-4. **Answer:** $\frac{40}{201}$

Accordingly as the unit's digit of the integer x is $0,1,2,3,4,5,6,7,8$ or 9, the unit's digit of x^2 is $0,1,4,9,6,5,6,9,4,1$. Of the 201 integers in $\{x \mid -100 \le x \le 100\}$, 40 have unit's digits of 1 or 9. Hence the number of integers in this set, whose squares have a unit's digit of 1, is 40; and the required probability is $\frac{40}{201}$.

6-5. **Answer:** $x = \frac{3\pm\sqrt{5}}{2}$

Add: $x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

$$\begin{array}{r} x^4 - 4x^3 + 5x^2 - 4x + 1 = 0 \\ x^2 \qquad\qquad = x^2 \\ \hline x^4 - 4x^3 + 6x^2 - 4x + 1 = x^2 \\ (x-1)^4 = x^2 \end{array}$$

$\therefore\ (x-1)^2 = \pm x$, solving for reals, $x = \frac{3\pm\sqrt{5}}{2}$

6-6. **Answer:** 90

Six dice have only a single face showing, so they each contribute 1 to the sum. Twelve dice have two faces showing, so they each contribute $1+2=3$. Eight dice have three faces showing, so they each contribute $1+2+3=6$ to the sum. The total is $6\cdot 1 + 12\cdot 3 + 8\cdot 6 = 90$.