Bergen County Math League

Calculators Permitted

Good Luck to You  **Good Luck to All**

Contest #1 **2023-2024** **12 minutes** **Questions 1 & 2**

1-1. When I saw Madder, he was part way up a ladder. He went up 4 rungs, down 7 rungs, and up 10. That put Madder at the top of the ladder. Then he went down 9 rungs, up 3 rungs, and down 10. That put him at the bottom of the ladder, that is, with his feet on the ground. How many rungs has Madder's ladder?

1-2. If a, b, and c are three different numbers for which

$$a^3 + pa + q = 0,$$
$$b^3 + pb + q = 0, \text{ and}$$
$$c^3 + pc + q = 0.$$

Find the value of the sum of $a + b + c$.

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Contest #1 **2023-2024** **12 minutes** **Questions 3 & 4**

1-3. If the ratio of the lengths of the edges of a right-rectangular prism is 1:2:3, and the total surface area is 550 square units, find its volume in cubic units.

1-4. Quarter-circles, centered at two opposite vertices of a square of area 16, and each having a radius of length 4, are drawn interior to the square. The diagonal of the square which joins these two vertices intersects the quarter-circles at E and F. If $EF = a - \sqrt{b}$, find the ordered pair of integers (a, b).

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Contest #1 **2023-2024** **12 minutes** **Questions 5 & 6**

1-5. The square of the difference of the roots of $ax^2 + bx + c = 0$, where $a \neq 0$, is equal to the square of their sum. Find c.

1-6. A ***lattice point*** in the plane is a point in the plane both of whose coordinates are integers. The line which is the graph of the equation $y - 1 = 2(x - 1)$ passes through an infinite number of lattice points, such as $(1, 1), (2, 3), (3, 5)$, etc. Find any real value of m for which the graph of the equation $y - 1 = m(x - 1)$ passes through the lattice point $(1, 1)$, but passes through no other lattice point in the plane.