Bergen County Math League
Calculators Permitted

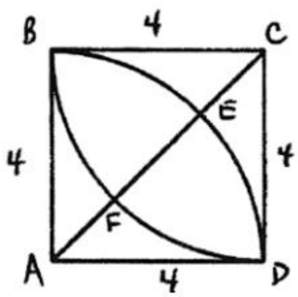
Contest #1 **2023-2024** **Answers/Solutions**

1-1. **Answer:** 16
Let rung 0 represent the ground, rung 1 the first rung on the ladder, and x the top rung on the ladder. Then x is also the number of rungs. From the second part of the journey, we have $x - 9 + 3 - 10 = 0$, so $x = 16$.

1-2. **Answer:** 0
By the nature of the given equations, a, b, and c are the three roots of $x^3 + px + q = 0$, so it must factor as $(x - a)(x - b)(x - c) = 0$. The coefficient of x^2 in this expansion is $-(a + b + c)$. But the coefficient of x^2 in this equation is 0, so $a + b + c = 0$.

1-3. **Answer:** 750
If x, y, and z are the edge lengths of the prism, then the surface area is $2(xy + xz + yz)$. But x, y, and z are in the ratio 1:2:3, and the surface area is 550, so we have: $2(x \cdot 2x + x \cdot 3x + 2x \cdot 3x) = 550$. This becomes $22x^2 = 550$, so $x = 5$. The volume is $x \cdot 2x \cdot 3x = 5 \cdot 10 \cdot 15 = 750$.

1-4. **Answer:** $(8, 32)$



$AC = AE + CF - EF$

$4\sqrt{2} = 4 + 4 - EF$

$\therefore EF = 8 - \sqrt{32} \Rightarrow (a, b) = (8, 32)$

1-5. **Answer:** 0
Let the roots be r and s. Then $(r + s)^2 = r^2 + 2rs + s^2$ and $(r - s)^2 = r^2 - 2rs + s^2$, so if $(r + s)^2 = (r - s)^2$, we must have $2rs = -2rs$, which means $rs = 0$. But the product of the roots of a $ax^2 + bx + c = 0$ is $\frac{c}{a}$, so $c = 0$.

1-6. **Answer:** any irrational value
Clearly the given equation passes through $(1,1)$ for any value of m. If (x_0, y_0) is any other lattice point on the line, then $m = \frac{y_0 - 1}{x_0 - 1}$ would be rational (as it is the ratio of integers). So if m is irrational, the line cannot pass through any other lattice point.