Bergen County Math League
Calculators Permitted

Contest #4 **2023-2024** **Answers/Solutions**

4-1. **Answer:** $\frac{1}{10}$

At least two but no more than three of the digits must be 1. If there are exactly two 1's, the other two digits must be 2, and there are $\binom{4}{2} = 6$ possible numbers. If there are three 1's, the other digit must be 3, and there are 4 possible numbers. Thus there are 10 possible numbers altogether, so there is a $\frac{1}{10}$ probability that a random guess is correct.
OR, simpler but less insightful explanation: The ten possible numbers are: 1122, 1212, 1221, 2112, 2121, 2211, 1113, 1131, 1311, 3111.

4-2. **Answer:** October 13

We begin with $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ seconds. To convert to minutes, divide by $10 \times 6 = 60$, leaving $9 \times 8 \times 7 \times 5 \times 4 \times 3 \times 2 \times 1$ minutes. Divide by $5 \times 4 \times 3 = 60$ again to see that this is $9 \times 8 \times 7 \times 2 \times 1$ hours. Finally, divide by 24 hours per day to leave $3 \times 7 \times 2 \times 1 = 42$ days. Starting at September 1, count 29 days to reach September 30, and then add another 13 days to reach October 13.

4-3. **Answer:** Day 14

She nets 4 feet upward during each day/night combination. So, after 13 days, she will have climbed 52 feet. On the next day, day 14, she climbs 11 feet, reaching the top of the tree (before climbing back down 7 feet).

4-4. **Answer:** 27

The right side $= 3 + \sqrt{2}$ and $8^{\frac{1}{6}} = \sqrt{2}$, so $x^{\frac{1}{3}} = 3$, or $x = 27$.

4-5. **Answer:** 101

$P = \left(\frac{2}{1}\right)\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\cdots\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n+1}\right)\cdots\left(\frac{100}{99}\right)\left(\frac{101}{100}\right) = 101.$

4-6. **Answer:** $-\frac{1}{2}$

Since the sum of any real number and its reciprocal ≥ 2 (or ≤ -2), each fraction must equal 1. Then, $\frac{x^2+2x+3}{x^2-2x+1} = 1 \Rightarrow x = -\frac{1}{2}$.

Alternatively, let $\frac{x^2+2x+3}{x^2-2x+1} = y$. Then, $y + \frac{1}{y} = 2 \Leftrightarrow y = 1 = \frac{x^2+2x+3}{x^2-2x+1}$, as before.