Bergen County Math League
NO Calculators Permitted

BCML

Contest #6 **2023-2024** **Answers/Solutions**

6-1. **Answer:** DAVE

One person took Dave's hat and Carol's coat; this person could not have been Dave or Carol, so it must have been Alice or Bob. But Alice took Bob's hat, so it must have been Bob who took Dave's hat and Carol's coat. Now Carol could not have taken Bob's hat (since Alice took it) or Dave's hat (since Bob took it), so she must have taken Alice's hat. The second clue then tells us that Alice was the owner of the coat taken by Dave.

6-2. **Answer:** 15

Note that once the number of four-packs and the number of three-packs are decided, all remaining item(s) must be packaged individually.
If three packages of four are used, the remaining three items allow for at most one three-pack, so the choice is whether to use one three-pack or not: **2 possibilities**
If two packages of four are used, there are seven items remaining, so either two, one, or zero three-packs can be used: **3 possibilities**
If one package of four is used, the remaining eleven items allow for three, two, one, or zero three-packs: **4 possibilities**
Finally, if no packages of four are used, all fifteen items are remaining, so the number of three-packs used can range from five to zero: 6 possibilities
In total, then, there are $2+3+4+6=15$ ways to package the order.

6-3. **Answer:** infinitely many, or ∞

$$\begin{aligned} a+b &= 6 \\ -b-c &= -12 \\ \underline{c+d} &\underline{= 17} \end{aligned}$$

Adding, $a+d=11$. Since the fourth equation is not independent of the other three equations, there are **infinitely many** solutions.

6-4. **Answer:** 60

Using the well-known expansion formulas,

$\sin 20^\circ + \sin x^\circ \cos 20^\circ - \cos x^\circ \sin 20^\circ = \sin x^\circ \cos 20^\circ + \cos x^\circ \sin 20^\circ$, or eliminating $\sin x^\circ \cos 20^\circ$ from both sides, yields: $\sin 20^\circ = 2\cos x^\circ \sin 20^\circ$.

Thus, $\cos x^\circ = \frac{1}{2}$, and the answer is 60.

6-5. **Answer:** $g(x) = \frac{3f(x)+1}{f(x)+3}$ or equivalent explicit **EQUATION**

$f(x) = \frac{x-1}{x+1}$ and $g(x) = \frac{2x-1}{2x+1}$. Then, $x = \frac{1+f(x)}{1-f(x)}$ and $x = \frac{1+g(x)}{2-2g(x)}$.

Hence, $\frac{1+f(x)}{1-f(x)} = \frac{1+g(x)}{2-2g(x)}$, and $g(x) = \frac{3f(x)+1}{f(x)+3}$.

6-6. **Answer:** 1

Note: In mathematics, an annulus is the region between two concentric circles.

Let r be the radius of the inner circle. Then the side length of the square is $2r$, so its diagonal is $2r\sqrt{2}$. The radius R of the outer circle is half the diagonal of the square, so $R = r\sqrt{2}$. The area of the annulus is $\pi R^2 - \pi r^2 = \pi\left(r\sqrt{2}\right)^2 - \pi r^2 = \pi r^2$, which is the same as the area of the inner circle.