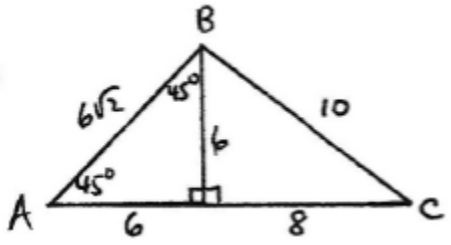
Bergen County Math League
Calculators Permitted

Contest #3 **2024-2025** **Answers/Solutions**

3-1. **Answer:** 2

1, 7, 49, and 343 are, respectively, $7^0, 7^1, 7^2$, and 7^3. Since no prize was awarded more than six times, the number of each prize that was awarded can be represented by a base-7 number. $1500_{10} = 4242_7 \Rightarrow$ the number of \$1 prizes is 2.

3-2. **Answer:** 42



Area = $\frac{(6+8)(6)}{2} = 42$

3-3. **Answer:** 11

$f(1) = a = 5 + 3 - 4 + 7 = 11.$

3-4. **Answer:** $1 + \sqrt{3}$

$\pi(r+2)^2 = 3(\pi r^2) \Rightarrow r^2 - 2r - 2 = 0.$ $\therefore r = \frac{2+\sqrt{12}}{2} = 1 + \sqrt{3}$

3-5. **Answer:** 3

$f(1) + f(2) = 10$

$-f(2) - f(3) = -9$

$f(3) + f(4) = 8$

$-f(4) - f(5) = -7$

$\vdots \quad \vdots$

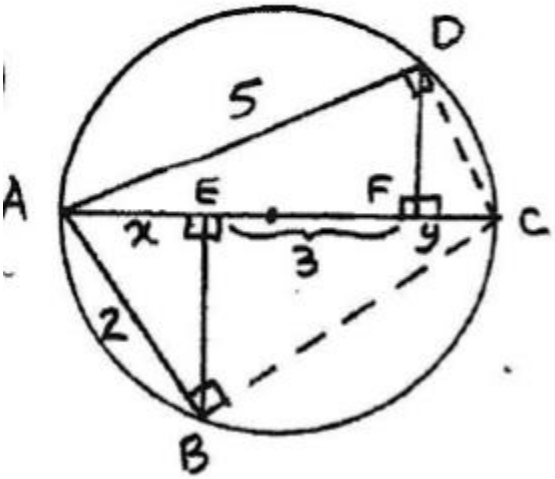
$-f(8) - f(9) = -3$

Adding, $f(1) - f(9) = 4$. But,

$\underline{f(1) + f(9) = 2}$

$2f(1) = 6$ and $f(1) = 3$.

3-6. **Answer:** 4



In right $\Delta ABC, AB^2 = (AE)(AC)$

In right $\Delta ADC, AD^2 = (AF)(AC)$

Thus, $4 = x(x + 3 + y) = x^2 + 3x + xy$

$25 = (x + 3)(x + 3 + y) = x^2 + 6x + 3y + xy + 9$

Subtracting, $21 = 3x + 3y + 9$, or $x + y = 4$.

Alternatively, dividing the two equations gives $\frac{25}{4} = \frac{x+3}{x}$, and from there one can solve for *x* and then *y*.