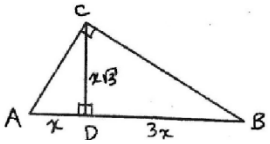
Bergen County Math League
Calculators Permitted

Contest #5 **2024-2025** **Answers/Solutions**

5-1. **Answer:** 48

To form a perfect square out of factors of x, we have to choose an even number of each of its prime factors. Considering first the prime factor 2, we can choose 0, 2, 4, or 6 of them, so we have 4 choices. Similarly, we have 3 choices for 3 $(0, 2, or\ 4)$, 1 choice for 5 (0), 2 choices for 7 $(0\ or\ 2)$, and 2 choices for 11 $(0\ or\ 2)$. Combined, we have $4 \times 3 \times 1 \times 2 \times 2 = 48$ choices.

5-2. **Answer:** 60



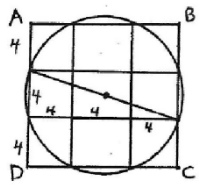
From the diagram, let $\mathrm{AD} = x$, $\mathrm{DB} = 3x$.

Thus, since $\mathrm{CD}^2 = (\mathrm{AD})(\mathrm{DB})$, $\mathrm{CD} = x\sqrt{3}$, and $m\angle \mathrm{DCB} = 60$

5-3. **Answer:** 2000

$f(1) = 2$. $f(2) = f(1) + f(1) = 2 \cdot f(1)$.
$f(3) = f(1) + f(1) + f(1)$ or $f(2) + f(1) = 3 \cdot f(1)$.
By mathematical induction, or otherwise, $f(n) = n \cdot f(1)$. Thus, $f(1000) = 1000f(1) = 2000$

5-4. **Answer:** 40



From the diagram, $d^2 = 16 + 144 = 160$.

Since $r^2 = \left(\frac{d}{2}\right)^2 = \frac{d^2}{4} = 40$, that is the answer

5-5. **Answer:** 33333333

Note that $4^8 = 100000000_4 = 33333333_4 + 1_4$. Subtract the 1. The answer is 33333333, with or without the subscript.

5-6. **Answer:** $\frac{13}{120}$ or $(13/120)^\circ$

The hour hand moves at a rate of $\frac{360^\circ}{12} = 30^\circ$ per hour, $\frac{30^\circ}{60} = \left(\frac{1}{2}\right)^\circ$ per minute, and $\frac{\left(\frac{1}{2}\right)^\circ}{60} = \left(\frac{1}{120}\right)^\circ$ per second.

At 8:43:37, then, the hour hand is at a position of $8 \times 30^\circ + 43 \times \left(\frac{1}{2}\right)^\circ + 37 \times \left(\frac{1}{120}\right)^\circ = \left(\frac{31417}{120}\right)^\circ$ relative to the "pointing up" position. Similarly, the minute hand moves at a rate of $\frac{360^\circ}{60} = 6^\circ$ per minute, and $\frac{6^\circ}{60} = \left(\frac{1}{10}\right)^\circ$ per second. At 8:43:37, it's at a position of $43 \times 6^\circ + 37 \times \left(\frac{1}{10}\right)^\circ = \left(\frac{2617}{10}\right)^\circ$.

The absolute difference between these positions is $\left(\frac{13}{120}\right)^\circ$.