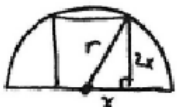
Bergen County Math League
NO Calculators Permitted

Contest #6 **2024-2025** **Answers/Solutions**

6-1. **Answer:** 5



$4x^2 = 20, x^2 = 5$
$r^2 = 5x^2 = 20 + 5 = 25 \Rightarrow r = 5$

6-2. **Answer:** $xyz = \frac{a-a^3}{3}$

Let $E = xy^2 + x^2y + xz^2 + x^2z + yz^2 + y^2z$.
Since $(x + y + z)(x^2 + y^2 + z^2) = x^3 + y^3 + z^3 + E$, then $E = a^3 - a$
But, $(x + y + z)^3 = x^3 + y^3 + z^3 + 3E + 6xyz$, or

$$a^3 = a + 3(a^3 - a) + 6xyz.$$

Thus, $xyz = \frac{a-a^3}{3}$

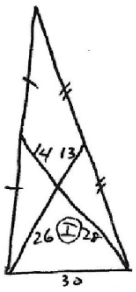
6-3. **Answer:** 0

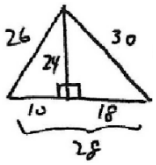
$a_0 = 2$
$a_1 = 4$

$$a_{0+2} = a_2 = \frac{3a_1 - 2a_0}{3} = \frac{12 - 4}{3} = \frac{8}{3}$$

$$a_{1+2} = a_3 = \frac{3a_2 - 2a_1}{3} = \frac{8 - 8}{3} = 0$$

6-4. **Answer:** 1008





$K_{26-28-30} = 36$ But, $K_I = \frac{1}{3}K_\Delta$ Note: bases are equal and the altitude is $\frac{1}{3}$.

$\therefore K_\Delta = 3 \times 336 = 1008$

6-5. **Answer:** 3:1

$$f\left(\frac{3x + x^3}{1 + 3x^2}\right) = \frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}} = \frac{1 + 3x + 3x^2 + x^3}{1 - 3x + 3x^2 - x^3} = \left(\frac{1 + x}{1 - x}\right)^3$$

Thus, $\log\left[f\left(\frac{3x+x^3}{1+3x^2}\right)\right] = 3\log\left(\frac{1+x}{1-x}\right) = 3\log[f(x)]$ and $\frac{y_2}{y_1} = 3:1$

6-6. **Answer:** 106

Units' Column: 25 times
Tens' Column: 30 times
Hundreds' Column: 51 times

$25 + 30 + 51 = 106$