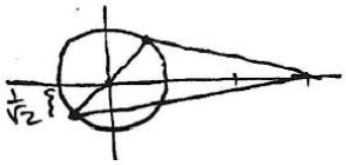
Bergen County Math League
NO Calculators Permitted

Contest #6 **2025-2026** **Answers/Solutions**

6-1. **Answer:** $(4,11)$
By inspection, the only possibility is $137 = 16 + 121 \Rightarrow (a, b) = (4,11)$

6-2. **Answer:** $\sqrt{10 + 3\sqrt{2}}$



By DeMoivre's Theorem, the two solutions of $z^2 = i$, i.e. the square roots of i, are $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ and $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. The latter, being in quadrant III, is further from 3 than the former (in quadrant I).

$$\left|3 - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)\right| = \sqrt{\left(3 + \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}} = \sqrt{10 + 3\sqrt{2}}$$

6-3. **Answer:** $10, \frac{1}{10}$
$(\log_{10} x)^2 = 1 \Rightarrow \log_{10} x = \pm 1 \Rightarrow x = 10, \frac{1}{10}$

6-4. **Answer:** $\frac{-3 \pm \sqrt{13}}{2}$
Let $y = x^2 + 3x + 1$

$\therefore y(y - 4) \leq -4 \Rightarrow (y - 2)^2 \leq 0 \Rightarrow y = 2$

$\therefore\ x^2 + 3x + 1 = 2 \Rightarrow x^2 + 3x - 1 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{13}}{2}$

6-5. **Answer:** 0
$\sin x < x$ for $x > 0$, so if $x > 0, (x + 1)^{\sin x} < (x + 1)^x$.

Only $x = 0$ works.

6-6. **Answer:** 0
Let m be the slope, so $m = \frac{y}{x+1}$, which is rational. We'll show x and y are both rational.
$1 = x^2 + y^2 = x^2 + m^2x^2 + 2m^2x + m^2 \Rightarrow 0 = x^2 + \frac{2m^2}{1+m^2}x + \frac{m^2-1}{1+m^2} = (x + 1)\left(x + \frac{m^2-1}{1+m^2}\right)$.
But $x \neq -1$, so $x = \frac{1-m^2}{1+m^2}$, which is rational.
Also, $y = \frac{2m}{m^2+1}$, so y is rational.
∴, there are 0 such points.